ART. XXIX.—*Ocean Waves and their Action on Floating Bodies.* By S. R. DEVERELL, ESQ.

[Read 11th September, 1871.]

1. According to Professor Rankine, the "energy of a wave is half actual, half potential." Whatever be the exact inference to be attached to this, it is plain that the power of ocean waves is the impressed force of the wind. The wind dies away, but its power still lives in the wave. Ocean waves, in fact, are vast reservoirs of power, or fly-wheels so to speak. When once set in motion, friction alone brings a wave to an end; theoretically, the motion would continue to be imparted from particle to particle for ever. How small the friction is, is evident from the distance to which waves visibly extend when a stone is dropped into a still lake; and it has been computed to be so small that the whole circumference of the earth would, in free water, be insufficient to destroy a wave sixty feet in height. Thus, in a calm at sea, we may witness huge rollers, known by seamen as a ground swell, progressing with undiminished force and velocity for weeks together, though there be no fresh impulse all the time; and the earthquake waves which receive their only impetus on the South American coasts, travel with destructive effect as far even as the Australian and Asiatic shores.

2. Until of late years, the theory of waves received very little attention, which fact is singular, considering the importance of the subject in ship-building, the construction of harbors, navigation, &c. (Art. Harbors, *Enc. Brit.* 8th ed.) Professor Airy refers the difficulty of mastering the subject to the imperfection of mathematics (Art. Tides and Waves Enc. Met.)* Even as late as the time of the construction of Eddystone lighthouse, Smeaton spoke of waves as being "amongst those powers of nature which admit of no calculation."

3. In contravention of Smeaton's opinion, is to be considered, that the production of waves is purely a dynamic process, and as such, a matter for calculation, as much as are the laws of falling bodies. In fact, the researches of Froude and Rankine have, despite the difficulties of the sub-

* For a similar remark, see an article on the Pendulum, by Mr. Sang, Enc. Brit. 8th Ed.

ject, placed it on too sound a basis for further controversy. They have shown that, amongst other properties,

4. The orbit of each particle of water in wave-motion is an ellipse; the form of which depends in a known law on the depth of water, so that in the ocean, the depth of which is approximately infinite, the orbit is circular.*

5. That the wave-surface is trochoidal in shape: (Rankine, *Manual of Applied Mechanics*' first published in 1858: "On exact form and motion of waves," *Trans. Roy. Soc.* 1862) such trochoidal profile being generated by rolling on the under side of a horizontal straight line, a circle whose radius is equal to the height of a conical pendulum, which revolves in the same period with the particles of liquid.

6. The hydrostatic pressure at each particle is the same as if the liquid were still. From these propositions, viz., 4, 5, and 6, which, for the sake of distinction, may be called Rankine's laws, there follow that—

7. If h be the height of a wave in deep water, its length is never less than πh.

8. The undulations of waves are performed in the same time as the oscillations of a pendulum whose length is equal to the breadth of the wave, or to the distance between two neighbouring cavities or eminences (see Art. Hydrodynamics *Enc. Brit.* 8th ed. p. 162.) Now the oscillations of pendulums being as the square roots of their radii, it hence follows that

9. The periods of waves are as the square roots of their lengths.†

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* In the propositions which follow, ocean-waves only are referred to. As in this paper frequent reference is made to previous passages, it has been found necessary to number them throughout.

† The length of the seconds pendulum at the Pole is 39·218 inches; at the Equator 39·018, whence the mean length = 39·118. Now the length of a pendulum whose period of oscillation is $t = 39{\cdot}118\,t^2$ in inches; consequently if the length of a wave $= b$ in feet, and v be its velocity in feet per second, we shall have $b = 3{\cdot}26\,t^2$ and $v = 3{\cdot}26\,t$; also $t = \sqrt{\frac{b}{3{\cdot}26}}$ from which equations all problems relating to cycloidal or breaking waves (which are those to which Smeaton's dogma more immediately referred) may be solved. If for b we substitute its value, $3{\cdot}1416\,h$, then $t = \sqrt{0.963\,h}$. Or finally—

$b = 3{\cdot}1916\,h.$

$t = 0{\cdot}3103\,\sqrt{h}$ where t = time the wave transverses space = its own length;

$v = 1{\cdot}0116\,\sqrt{h}.$

By which it appears that the velocity of a cycloidal wave per second is very nearly = square root of its height in feet.

10. With regard to the origin of ocean-waves, Professor Airy observes, "it is to be understood that either from preceding disturbances or from trifling irregularities in the action of the wind while the water is smooth, there are very shallow undulations on the water.

Now, with all due deference to so great an authority, this surely attributes these stupendous and invariable results primarily to an accident. For, be it observed, every wave, no matter how enormous, must have had origin in a primary wavelet: to say it started into being at any other stage, is merely to enlarge the supposed magnitude of such first wavelet. In fact, the word regularities might be justly substituted for irregularities in the above remark; for the Professor's own mode of demonstration shows how the force of the wind acting in the direction of a tangent to the surface may produce the initial wavelet.

Let *a b (fig.* 1, *plate* 1*)* be the smooth surface of water, *c* a particle of a column of air moving in the direction of the arrow; *d e* a stationary column of air. When the breeze commences, the particle *c* is pressed against the particle *d*, which being stationary, reacts the pressure in all directions, shown by the dotted lines. Thus, a downward pressure is produced, and also a retrogressive pressure in the resulting hollow, in accordance with the theoretical motion of the particles of water; while the particle *c* being reinforced, the column *d e* yields, and the onward motion of the wavelet takes place.

Regarding the wind as a constant force, it now increases the result, viz., the initial wavelet, in volume and velocity. But this acceleration in the magnitude of the wave has its practical limit, for the wave recedes from the advancing air, and the wind, although a constant, being thus also a following force, its *impulsive* effect on the wave becomes retarded with the increase in the magnitude, and therefore, the velocity (9) of the wave; so that, if the maximum force of the wind be known, the maximum dimensions of waves may be ascertained. The observed results agree in the main with theory. Were it not for the cause stated, it is obvious that, in a continuous gale of wind, waves would go on increasing to indefinite dimensions.

11. The force resident in a wave then is to be considered not as the received strength of the wind simply, but as the *aggregated* impulse from the time of its commencement. The wave becomes in fact a storehouse of power, retaining,

Fig 1

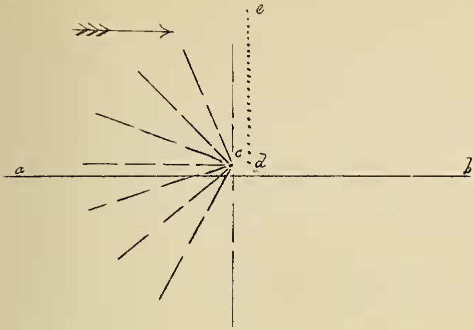


Fig 2

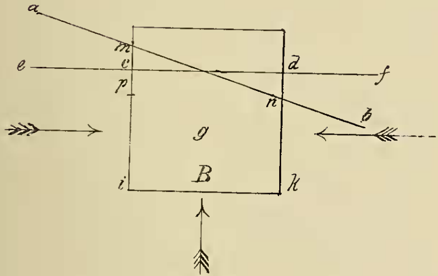
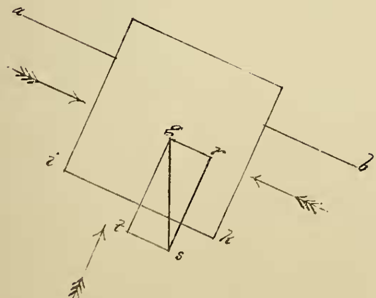


Fig 3.



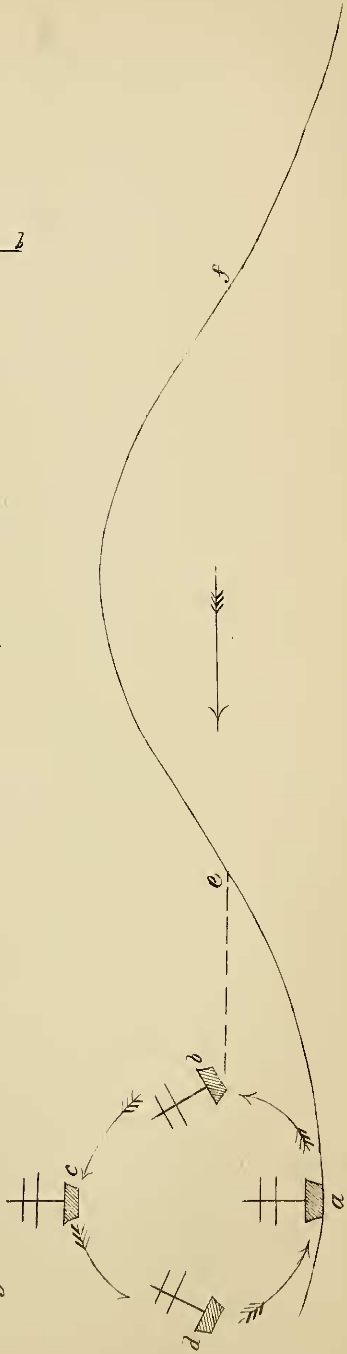


Fig 5

like a vast fly-wheel, the acquired momentum of the air. Assuming the great forces which set the atmosphere in motion to be derived from heat, we may conceive the sun to be the prime source of power, and the ocean probably as the mightiest reservoir of its force upon earth. There is no cessation of its giant activity, and it may without error be said, that from the time an ocean vessel leaves one port to the time she enters the next, she is heaving upon perpetual waves.

12. To instance the force thus aggregated in waves, there is mentioned that at Port Sonachan in Loch Awe, with the diminutive fetch of only 13 or 14 miles, a stone weighing $\frac{1}{4}$ ton was torn out of the masonry of the landing slip, and overturned by a wave (*Enc. Brit.* Art. Harbors, 8th ed.), and a stone $\frac{1}{2}$ ton weight was similarly moved and overturned at Buffalo (Stevenson's *Engineering of North America*). At Barrahead, one of the Hebrides, a wave was seen to strike a block of granite of 50 tons weight, and move it several feet. In November, 1817, the waves of the German Ocean, where the fetch is 600 miles, overturned, just after it had been finished, a column of freestone 36 feet high, 17 feet base, and at the point of fracture 11 feet in diameter (*Enc. Brit.* Art. Lighthouses). The prodigious impulse requisite to do this is shown in the height to which waves ascend against cliffs and headlands. Such effects are witnessed in their utmost grandeur in the islands on the west coast of Scotland, on the sides of icebergs, and at many points on the South African and Australian shores. So also at lighthouses; at Bell Rock and Skerryvore, waves are known to dash completely over the summit of the edifices; and a magnificent spectacle is presented at Cape Northumberland in South Australia, where a massive rock, said to be from 70 to 100 feet in height, is swept by every sea during a gale of wind.

13. In the German Ocean, with a fetch of 600 miles, the height of waves varies from 13 to 20 feet: the Mediterranean with the same fetch, gives the same results. But these proportions sink into insignificance in comparison with the great mid-ocean waves of the Atlantic and South Pacific. Dr. Scoresby (Brit. Assoc., 1850) states, the maximum height of waves off the Cape of Good Hope to be 43 feet: other writers make it considerably more, and it has even been estimated that, from summit to summit of these enormous masses of water, a distance of nearly the fourth

of a mile may sometimes intervene.* In the presence of such colossal forces, a vessel is like a toy ; nor on reflection is it less than wonderful that an object comparatively so fragile, can make her way amidst them unchecked and unharmed.

14. By an instrument called the Marine dynamometer, the force of waves at Bell Rock (North Sea) has been ascertained to be 1½ tons per square foot, and at Skerryvore lighthouse in the Atlantic, 3 tons per square foot. That is, if a plate a foot square were inserted in the summit of a wave, these pressures would be exerted on it. A practical example of this gigantic force is shown in those occurrences, often disastrous enough, when a ship is "struck" by a sea, that is, when she meets a wave without rising to it. On such occasions, the vessel seems to hang motionless for an interval as if stunned, every bolt and timber quivering from stem to stern. In running or nearly so before a wind, a ship so struck is liable to be forced round by the huge impulse, and if there be great way upon her, to broach-to, to the almost certain destruction of a spar or spars. Another frequent occurrence is that known as "settling," common to ships laden with shifting or loose cargo. A fleet of sugar ships coming in port sometimes presents a ludicrous appearance from this cause, the masts of the vessels inclining in various angles and directions.

An instance of this kind was witnessed by the writer, in the China seas, in 1851. During the prevalence of a dead calm, accompanied however with enormous rollers or ground swell, the result of a typhoon a week previously, the vessel —which was one of 1,200 tons—having no headway, was caught

* Latterly, in 1869, waves have been measured *in the English Channel*, with a height of 43 feet; it is probable, therefore, that Dr. Scoresby's estimate for deep ocean waves is sometimes greatly exceeded. The reference to the measurement here stated has unfortunately beon mislaid. Maury, who, in his exhaustive work on the Physical Geography of the Sea, does not treat distinctively of the waves, yet refers in enthusiastic terms to those of the great Southern Ocean. "To appreciate, he says, the force and volume of these polar bound winds in the Southern Hemisphere, it is necessary that one should 'run them down' in that waste of waters beyond the parallel of 40° south, where 'the winds howl and the seas roar.' The billows there lift themselves up in long ridges with deep hollows between them. They run high and fast, tossing their white caps aloft in the air, looking like the green hills of a rolling prairie capped with snow, and chasing each other in sport; still their march is stately and their roll majestic. The scenery among them is grand, and the Australian-bound trader after doubling the Cape of Good Hope, finds herself followed for weeks at a time by these magnificent rolling swells, driven and lashed by the 'brave west wind' furiously."—*Physical Geography of the Sea*, p. 361.

broadside on one of these waves with such force as caused the cargo, consisting for the greater part of sugar, to settle bodily on the port side. Ascending the next undulation, the loosened mass settled with still greater violence on the starboard side and was again thrown over to the port side, where it remained during the voyage, the vessel retaining a port list of about 15°. All this may be said to have taken place during the passage of one wave.

17. To arrive at the nature and extent of the wave forces affecting a floating body, it is necessary to investigate the *theoretical* motions of such a body. For this purpose, suppose the body to be of perfect buoyancy, that is a body which, its magnitude being inconsiderable with respect to that of the wave, possesses equal mobility with the particles of water on which it rests, so that it accompanies the particles in their motion. This it can only do, if absolutely inconsiderable, but we may suppose a very small object, such as a cork for instance, to approximate towards the motion of a particle of water. From the laws thus deduced, we shall be able to obtain modified views with respect to floating bodies of imperfect buoyancy, that is, of bodies which, being large with respect to the wave, possess unequal mobility with the particles of liquid on which they rest, and which therefore move relatively to them.

18. The direction of the force of buoyancy is always at right angles to the nearest surface of the water, whether that surface be horizontal, as in the case of smooth water, oblique, as in a descending stream, or variable, as in waves.*

Thus, let (*fig.* 2, *plate* 1) *e f* represent the surface of smooth horizontal water, *B* a floating body in it. Now, the water pressing upon all sides of the body, as shown by the arrows, the lateral pressures on the sides *c i* and *d k* counteract each other, and the upward force of buoyancy is counteracted by and is equal to the weight of *B*. But, suppose the water became a descending stream whose inclined surface is *a b*; the lateral pressures are now *m i* and *n k*; take *p i* equal to *n k*, then the preponderating lateral pressure *m p* will turn

* The wording of Archimedes' law, viz.: "that if a solid body be either wholly or partially immersed in a liquid, it is pressed upwards with a force which is equal to the weight of the liquid displaced, and whose point of application is the centre of gravity of the displaced liquid" is incorrect, if by upward is meant vertically upward. A cork, for instance, immersed in a wave, will proceed to the nearest point in the wave surface, not necessarily in a vertical direction.

the body round upon its centre of gravity g, until the lateral pressures or immersions of the sides are equal ; that is until $i\ k$ is parallel to $a\ b$.

The body then assumes the position shown in *(fig.* 3, *plate* 1) Let $g\ s$ be a vertical line representing the weight of the body : this is equivalent to two forces $g\ r$ parallel with, and $g\ t$ at right angles to the surface of the liquid $a\ b$: consequently, $t\ g$ at right angles to the surface will represent the force of buoyancy.* Hence by parity of reasoning the force of buoyancy acts everywhere at right angles to the surface of wave, and therefore,

19. A body floating on a wave will assume a position at every point such that the plane of flotation will be parallel to the wave surface at that point : the deviation of the mast from the vertical being equal to the angle which the tangent to the trochoidal surface at that point forms with the horizon. Thus, *(fig.* 4, *plate* 2*)* different objects placed in the five points, $a\ b\ c\ d\ e$ will assume the angular positions shown. And one object, suppose that at a, will, during the passage of a wave, assume these positions successively. For from the definition (17)—

20. A body floating on a wave travels in the same circular orbit as the surface particles on which it rests. *(Fig.* 5, *plate* 1.*)* Suppose the wave proceed from right to left, as shown by the large arrow. During its passage the body at a performs the circle $a\ b\ c\ d$, and at the same time has an angular motion or oscillation about its centre of gravity, such that the plane of flotation is everywhere a tangent to the surface; for instance, at b and d the plane is tangential to the curve at e and f on the same level. The velocity with which the body is carried round is of course uniform, and in deep water is always less than the horizontal velocity of the wave.

21. The absolute motion or locus described by a given point of the floating body is an epicycloidal curve resembling an ellipse, one of whose axes is the diameter of the orbital circle. For instance, *(fig.* 6, *plate* 2*)* if $a\ b$ $c\ d$ be the orbital circle as before, the mast-head will

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* Hence it appears that the common proposition that a floating body displaces its own weight of water is incorrect, *except* in the case when the the surface of the water in which it floats is horizontal. If θ represent the angle of inclination of the surface, and B the weight of the body, $B \cos \theta$ is the true weight of the displaced water. Hence also, a body on the side of a wave, is less immersed than when in the trough or on the summit.

Fig. 4.

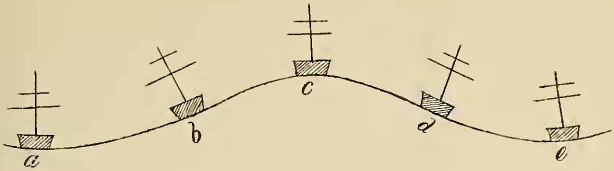


Fig 6.

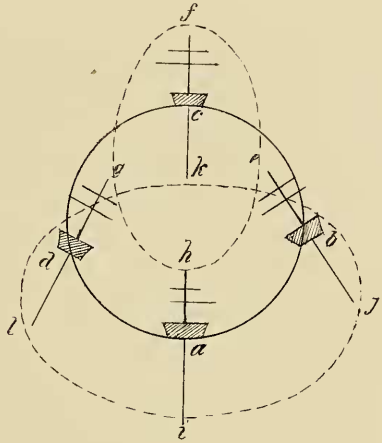


Fig 7

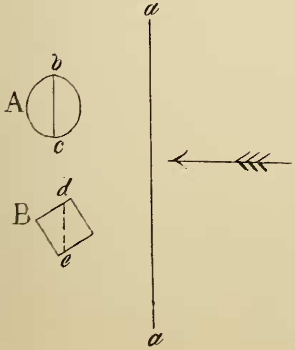
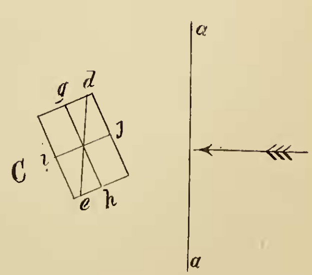


Fig 8



describe approximately the ellipse $h\ e\ f\ g$. The diameter of the orbital circle is here the major axis, the given point, viz. the mast-head, being *above* the centre of gravity of the body. But if it be *below*, as suppose by the projection of the mast beneath, the ellipse $i\ j\ k\ l$ is described, in which the diameter of the orbital circle $k\ i$ is equal to the *minor* axis. The other axis is the distance between the two positions which the given point occupies, at the approaching and departing mid-points of the wave.

22. The axis of the angular motion is parallel to the ridge of the wave.

Fig. 7. Let $a\ a$ represent the ridge of the wave advancing in the direction shown by the arrow; A and B two floating bodies. Now, as the solid form of the wave may be supposed to be generated by the motion of the trochoidal surface curve along the straight line of the ridge, the water is level at every point of the diameter $b\ c$ of the body A drawn parallel to $a\ a$; the force of buoyancy therefore affects the parts on either side of this line, whence $b\ c$ is the axis of angular motion. Similarly, in the cubic body B, the line $d\ e$ passing parallel to $a\ a$ through the centre of gravity of the body, is the axis of the angular motion of B. For the sake of distinction call this axis of motion, parallel to the ridge, the resultant or wave axis: and the inclination upon it the wave angle.

Fig. 8. If the body be an oblong figure of inconsiderable dimensions, as at C, and it be required to refer the angular motion on the wave axis $d\ e$ to angular motions on the axis $i\ j$ and $g\ h$ of the body, in order to determine the oblique position of the body with respect to the ridge of the wave, call $i\ j$ the pitch axis, $g\ h$ the rolling axis. Let the whole angle of inclination on the wave axis be θ; the required inclinations on $g\ h$ and $i\ j$, or the rolling and pitch angles respectively equal to ϕ and ψ; then, by a well-known theorem of spherics $\cos\theta = \cos\phi\cos\psi$.

If a plummet be suspended from the mast, the angle between the mast line and plummet line is equal to the wave angle.

23. The greatest angle of inclination of the body is at the mid point of the wave, and is less than half a right angle. In the figure 5, paragraph 20, let b be the mid-point on the circle corresponding to the position e on the wave. Now, it is a property of this curve that a tangent at the point e is parallel to a chord $d\ c$ of the rolling circle, and this chord

being that of 90° it forms with the horizontal line *d b e* an angle of 45°, which is therefore the maximum slope angle at the point *e*.*

24. The whole motion of the body is compounded of three *oscillations*, viz.: a bodily horizontal oscillation, or sway, thus  ; a bodily vertical oscillation, or heave, thus  ; and an angular oscillation, or roll, thus  around the wave axis or centre of gravity.

In the upper semicircle (figure 5, paragraph 20) the bodily sway is from right to left, the angular motion of the mast from left to right, and these are reversed in the lower semicircle. Hence we have for general rules that—

When a body rises on a wave it is swayed horizontally first against the wave and then with it, but when falling on a wave, first with it and then against it.

A floating body on the upper part of a wave is swayed *with* the wave; on the lower part against it.

The sway is equal to the heave or vertical rise and fall on approach and departure of the wave.

During the passage of a wave, there are two oscillations of the plummet equal to twice the wave angle; or four half oscillations equal to the wave angle.

These laws are universal, for though they are deduced on the assumption that the floating body is inconsiderable in magnitude with respect to the wave, they hold in a modified degree in the case of large bodies. For instance, if one end of a vessel rises to a wave, it is certain that the *part* which rises sways first against the advancing wave and then with it, and in falling, first follows the receding wave and then leaves it.

25. A ship, one diameter of whose plane of flotation is greater than the other, buoyed upon a wave or different parts of a wave, is acted on variously by their forces, which, as the vessel is a rigid body, in a great measure neutralise each other; her actual motion of course is in accordance with the

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* By *slope angle* in future will be intended the greatest inclination or *maximum* angle attained by a vessel of any magnitude during an oscillation. It is customary for shipwrights to denote this angle by θ as above.

Fig. 9.

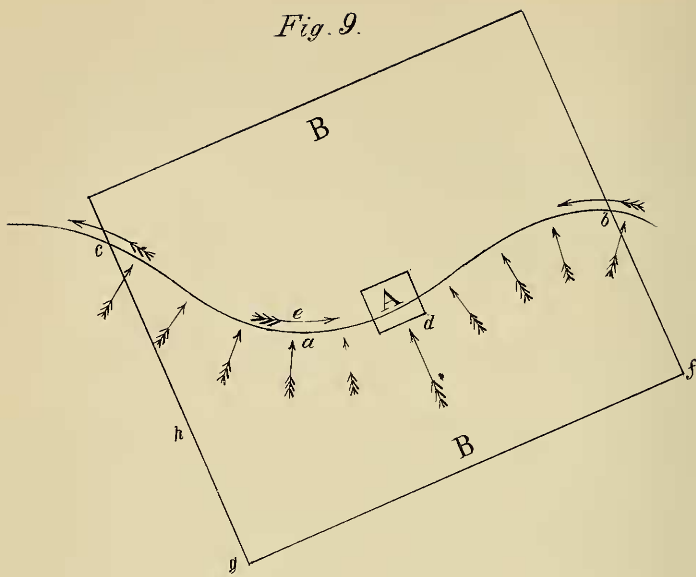
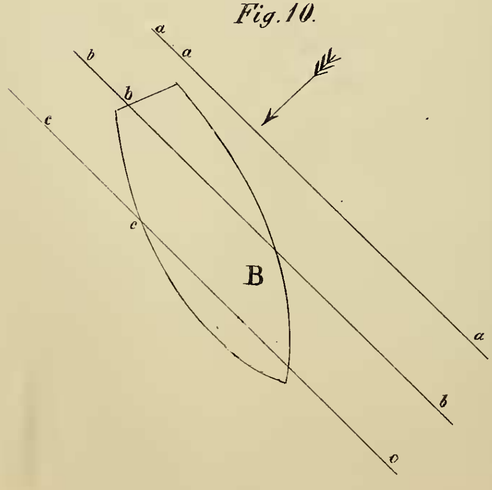


Fig. 10.



resultant of these forces. Hence the angle of inclination which she assumes will determine her position upon the resultant wave.

Fig. 9. For instance, suppose *A* be a small body upon the lower part of a wave, *a b* progressing from right to left. *A* being inconsiderable the force of buoyancy is in the single direction shown by the arrow *d* (18), and being in the lower part of the wave, the body partakes singly of the moving force of the wave in the retrogade direction, shown by the arrow *e* (22). Now, suppose a very large body *B B* of similar figure to *A*, and whose centre of gravity is in the same point, be held in a similar position to *A* as shown in the figure. The body *B* covers the whole wave surface *c a b*, and is acted on by the force of buoyancy in all the various directions shown by the small arrows at right angles to the surface; the resultant of which is such that *B* cannot maintain the same position as *A* ; for resolving the force of buoyancy into the hydrostatic pressure on the sides *b f*, *f g* and *g c*, and taking *g h* equal to *b f*, the force of buoyancy, that is the pressure of the water to fill up the displacement, acts on the sides *f g* and *h c* (since *b f* and *h g* neutralise each other) will force the body round by virtue of the preponderating force on *h c*. Again, since *B* covers the summit as well as the trough, the moving force of the wave urges it on at the summit and backward in the trough (23). On account of these conflicting actions, the body *B* moves only partially with the particles of water in which it floats, and therefore as the positions of the waves constantly shift with respect to it, the motion of a large body floating on waves may be described as a ceaseless attempt to obtain an equilibrium which it never acquires.*

26. *Fig.* 10. But, whatever be the composite motion of the body *B*, this motion will be regular, that is it will be *regularly repeated* on the same system of waves. For, suppose the body *B* rest upon a series of parallel waves *a a*, *b b*, *c c*, progressing in the direction shown by the arrow, and in the position shown, the mast will assume an angle with the vertical equal to θ. Then the waves being uniform when *b* has progressed to *c*, *a* has progressed to *b* ; the relative position on the waves being similar; consequently, as like causes produce like effects, the mast will assume the same angle with the vertical as before. Nor will the result be affected

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* See the footnote, § 46.

if the vessel be moving uniformly on her course : the periods of the oscillations only will be affected : her angular position with respect to the ridges remaining the same.

Fig. 11. Next, suppose the vessel be acted on by two systems of waves, viz., *a a, b b, c c,* as before, and *p p, q q, r r, &c.,* progressing obliquely to them : and in the position denoted the mast assume an angle with the vertical equal to θ. Now, if the system *a a* progress with the same velocity as the system *p p* (in which case they must be equal in magnitude {8}); when the wave *a* has progressed to the position *b, p* has progressed to the position *q,* and the two systems are in the same position as before with respect to the vessel: whence the same inclination θ will be repeated with the passage of each wave.

If, however, the velocity of one system be *n* times that of the other, *n* of the large (and therefore more rapid) waves will have to pass before the next less one brings the two systems into the same relative position with the body. In this case therefore the vessel will repeat the same cycle of movements ; hence, universally, if a floating body however great float upon two or more systems of waves, and keep the same course or angle with respect to them, her motions upon them will be performed in regular cycles, each cycle comprising as many passages of the largest system of waves as is equal to the least common multiple of the velocities of all the systems.

27. The wave motions of a vessel of any size, steering a uniform course at sea, therefore, instead of being irregular follow these exact laws. In fact, we may imagine a resultant wave and the position and oscillations of a floating body of any magnitude or shape to be referable to those of an *inconsiderable* body upon this resultant wave. The orbit of the particles in such a wave would be elliptical, the elements of the wave being peculiar to each vessel, according to her magnitude and build. These elements can be accurately obtained by observations on bodies entirely withinboard, as will be subsequently shown. The period, dimensions, and position of the hypothetical wave being thence deducible.

Of course there are disturbing influences, such as immediate effects of the wind, variations in the wave elements, in the vessel's course, &c. Making allowances for these, however, a long series of experiments would furnish reliable if not very exact data for measurement of the mean elements of resultant waves.

Fig. 11.

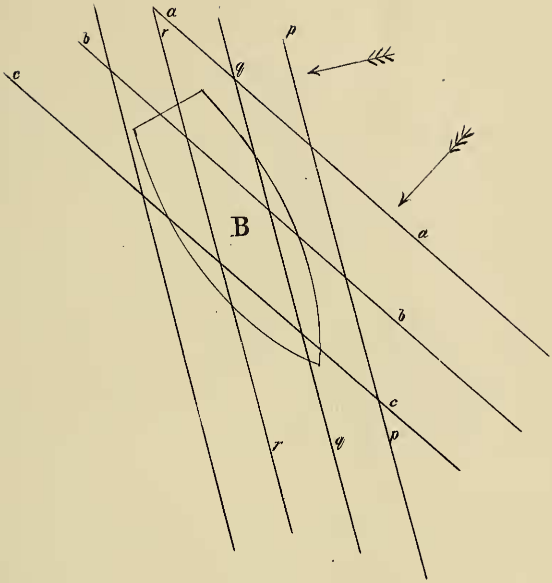

Fig 12